Exam Total/120: Quiz Ave:	Name:
Exam Percent: Course Ave:	

Spring 2016 Math 245 Main Midterm

Please read and follow these directions:
Please write legibly, with plenty of white space. Please print your name in the designated box, similarly to your quizzes. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will last 75 minutes; pace yourself accordingly. Please try to ensure a quiet test environment for others. Good luck!

Problem 1. Carefully state the "Division Algorithm" theorem.

Problem 2. Prove that the square of a rational number is rational.

Problem 3. Let $n \in \mathbb{Z}$. Prove that $\lceil \frac{n}{2} \rceil \geq \frac{n-1}{2}$.

Problem 4. Carefully define each of the following terms:

a. nand

b. hypothetical syllogism

c. constructive existence proof

d. $\lfloor x \rfloor$

e. proof by contradiction

Problem 5. Carefully define each of the following terms:

a. strong induction

b. $a|b$ $(a, b \in \mathbb{Z})$

c. $A \subseteq B$

d. $A \cap B$

e. $|A|$ (A is a set)

Problem 6. Prove or disprove: $\forall x \in \mathbb{R}, \exists y, z \in \mathbb{R},\ y^2 < x^2 < z^2$.

Problem 7. Give a mathematical statement with one free variable and two bound variables.

Problem 8. A Boolean algebra is a nonempty set S, two binary operations $\oplus, \odot$, and six axioms. Carefully state any three of these axioms.

Problem 9. Let A, B be sets. Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

Problem 10. Prove that $\sqrt{3}$ is irrational.

Problem 11. Use the extended Euclidean algorithm to find $\gcd(56, 133)$ and to find integers x, y so that $\gcd(56, 133) = 56x + 133y$.

Problem 12. Use induction to prove that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for all $n \in \mathbb{N}$.